

Semiconductor Engineering

:Semiconductor Physics and Devices

Chapter 1. Semiconductor Electronics

Objectives

Overview of the physical electronics of semiconductor

- Energy band theory
- Doping principle
- Free carrier statistics
- Drift and diffusion

Thermal-Equilibrium Statistics

For more discussion of electrical conduction in a semiconductor

Additional concept should be considered;

First, the **concept of thermal equilibrium**;

Second, the relationship at **thermal equilibrium between the majority- and minority-carrier concentrations** in a semiconductor;

Third, the use of Fermi statistics and the **Fermi level to specify the carrier concentrations**.

Thermal-Equilibrium Statistics

Thermal Equilibrium

free-carrier densities in semiconductors are related to the populations of allowed states in the conduction and valence bands

Although a semiconductor crystal can be excited by external sources of energy such as incident photoelectric radiation, many situations exist where the total energy is a function only of the crystal temperature

➡ In this case the semiconductor spontaneously (but not instantaneously) reaches a state known as *thermal equilibrium*.

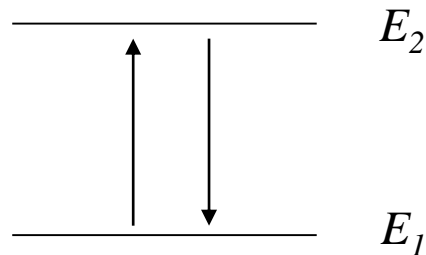
➡ **Thermal equilibrium** is a dynamic situation in which every process is balanced by its inverse process

Thermal-Equilibrium Statistics

Thermal Equilibrium

Example

at thermal equilibrium, if electrons are being excited from a lower energy E_1 to a higher energy E_2 , then there must be equal transfer of electrons from the states at E_2 to those at E_1



Likewise, if energy is being transferred into the electron population from the crystal vibrations (phonons), then at thermal equilibrium an equal flow of energy is occurring in the opposite direction.

Thermal-Equilibrium Statistics

Mass-Action Law.

At most temperatures of interest to us, there is sufficient thermal energy to excite some electrons from the valence band to the conduction band.

A **dynamic equilibrium** exists in which some electrons are constantly being excited into the conduction band while others are losing energy and falling back across the energy gap to the valence band.

The excitation of an electron from the valence band to the conduction band corresponds to the generation of a hole and an electron, while an electron falling back across the gap corresponds to electron-hole recombination

Thermal-Equilibrium Statistics

Mass-Action Law.

The generation rate of electron-hole pairs G depends on the temperature T but is, to first order, independent of the number of carriers already present.

$$G = f_1(T)$$

$f_1(T)$ is a function determined by crystal physics and temperature

The rate of recombination R

depends on the concentration of electrons n in the conduction band and also on the concentration of holes p (empty states) in the valence band, because both species must interact for recombination to occur.

$$R = npf_2(T)$$

Thermal-Equilibrium Statistics

Mass-Action Law.

At equilibrium

the generation rate must equal the recombination rate. $\mathbf{G} = \mathbf{R}$, in Equations

$$npf_2(T) = f_1(T)$$

$$G = f_1(T) \quad R = npf_2(T)$$

$$\rightarrow np = \frac{f_1(T)}{f_2(T)} = f_3(T)$$

n ; concentration of electrons

p ; concentration of holes

the product of the hole and electron densities (np) in a given semiconductor is a function only of temperature

Thermal-Equilibrium Statistics

Mass-Action Law.

Intrinsic semiconductor (undoped) case

all carriers result from excitation across the forbidden gap.

$\underline{n = p = n_i}$ where the subscript i reminds us that we are dealing with intrinsic material

$$np = \frac{f_1(T)}{f_2(T)} = f_3(T) \longrightarrow n_i p_i = n_i^2 = f_3(T)$$

The intrinsic carrier concentration depends on temperature because thermal energy is the source of carrier excitation across the forbidden energy gap.

The intrinsic concentration is also a **function of the size of the energy gap** because fewer electrons can be excited across a larger gap.

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

N_c and N_v are related to the density of allowed states near the edges of the conduction band and valence band

Thermal-Equilibrium Statistics

Mass-Action Law.

Intrinsic semiconductor (undoped) case

Because the intrinsic carrier concentration n_i is constant for a given semiconductor at a fixed temperature, it is useful to replace $f_3(T)$ by n_i^2 in $n_i p_i = n_i^2 = f_3(T)$

$$\rightarrow np = n_i^2$$

it shows that increasing the number of electrons in a sample by adding donors causes the hole concentration to decrease so that the product np remains constant.

Similarly, This result, often called the *mass-action law*, has its counterpart in the behavior of interacting chemical species, such as the concentrations of hydrogen and hydroxyl ions (H^+ and OH^-) in acidic or basic solutions

Thermal-Equilibrium Statistics

Mass-Action Law.

In the neutral regions of a semiconductor (i.e., regions free of field gradients)

The number of positive charges must be exactly balanced by the number of negative charges

Positive charges exist on ionized donor atoms and on holes, while negative charges are associated with ionized acceptors and electrons.

→ If there is charge neutrality in a region where all dopant atoms are ionized

$$N_d + p = N_a + n$$

↑
————— $np = n_i^2$



$$n - \frac{n_i^2}{n} = N_d - N_a$$

the electron concentration n :

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2} \right)^2 + n_i^2 \right]^{1/2}$$

Thermal-Equilibrium Statistics

Mass-Action Law.

In an n-type semiconductor $N_d > N_a$

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2} \right)^2 + n_i^2 \right]^{1/2}$$

the electron density depends on the net excess of ionized donors over acceptors

a piece of *p-type* material containing N_a acceptors can be converted into n-type material by adding an excess of donors so that $N_d > N_a$

For example

For silicon at room temperature, n_i is $1.45 \times 10^{10} \text{ cm}^{-3}$

the net donor density in n-type silicon is typically about 10^{15} cm^{-3} or greater

$$(N_d - N_a) \gg n_i$$

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2} \right)^2 + n_i^2 \right]^{1/2}$$

$$\underline{n} \approx (N_d - N_a)$$

$$np = n_i^2$$

$$p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_d - N_a}$$

Thermal-Equilibrium Statistics

Mass-Action Law.

$$p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_d - N_a}$$

When: n_i is $1.45 \times 10^{10} \text{ cm}^{-3}$

$$N_d - N_a = 10^{15} \text{ cm}^{-3}$$

The calculated $p = 2 \times 10^5 \text{ cm}^{-3}$

the minority-carrier concentration is nearly 10 orders of magnitude below the majority-carrier population

In general, the concentration of one type of carrier is many orders of magnitude greater than that of the other in extrinsic semiconductors.

Thermal-Equilibrium Statistics

Fermi Level

The numbers of free carriers (electrons and holes) in any macroscopic piece of semiconductor are relatively large-usually large enough to allow use of the laws of statistical mechanics to determine physical properties.

Fermi-Dirac distribution function

$$f_D(E) = \frac{1}{1 + \exp[(E - E_f)/kT]}$$

E_f is a reference energy called the *Fermi energy* or *Fermi level*

At $E = E_f$, $f_D(E_f)$ always equals 1/2.

The Fermi-Dirac distribution function, often called simply the *Fermi function*, describes **the probability that a state at energy E is filled by an electron.**

Thermal-Equilibrium Statistics

Fermi Level

- ➔ The Fermi-Dirac distribution function, often called simply the *Fermi function*, describes **the probability that a state at energy E is filled by an electron**.

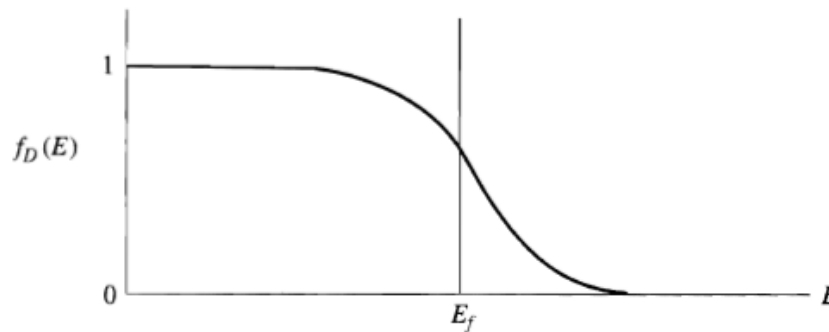


FIGURE 1.10 (a) Fermi-Dirac distribution function describing the probability that an allowed state at energy E is occupied by an electron.

the Fermi function approaches unity at energies much lower than E_f , indicating that the lower energy states are mostly filled.

It is very small at higher energies, indicating that few electrons are found in high-energy states at thermal equilibrium—in agreement with physical intuition.

At absolute zero temperature all allowed states below E_f are filled and all states above it are empty. At finite temperatures, the Fermi function does not change so abruptly; there is a small probability that some states above the Fermi level are occupied and some states below it are empty.

Thermal-Equilibrium Statistics

Fermi Level

Fermi-Dirac distribution of electrons in solid

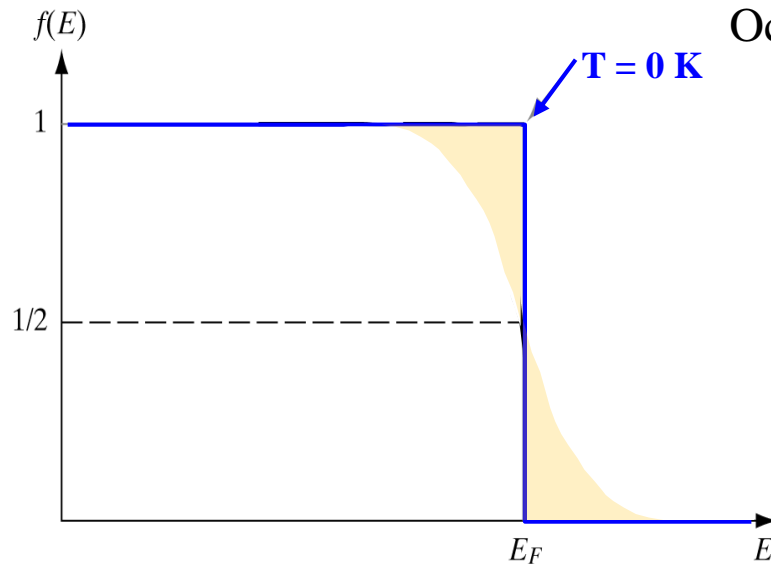
$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

Distribution of electrons over a range of allowed energy level at thermal equilibrium.

→ Probability that an available energy state at E will be occupied by an electron at absolute temperature T .

k : Boltzmann's constant (8.62×10^{-5} eV/K)

E_F : Fermi level (Energy that the occupation probability by an electron is exactly 1/2.)



Occupation probability at Fermi level → $f(E_F) = 1/2$

@ $T = 0$ K

i) $E < E_f \rightarrow f(E) = 1$

ii) $E > E_f \rightarrow f(E) = 0$

@ $T = T$ K

i) $(E - E_f) > 3kT \rightarrow f(E) = e^{-(E - E_f)/kT}$: electron

ii) $(E - E_f) < 3kT \rightarrow f(E) = 1 - e^{-(E - E_f)/kT}$: hole

$$*e^3 \sim 20, e^{-3} \sim 0.05$$

Thermal-Equilibrium Statistics

Carrier concentration in intrinsic semiconductor

The Fermi function represents only a probability of occupancy. It does not contain any information about the states available for occupancy and, therefore, cannot by itself specify the electron population at a given energy

Applying quantum physics to a given system provides information about the density of available states as a function of energy

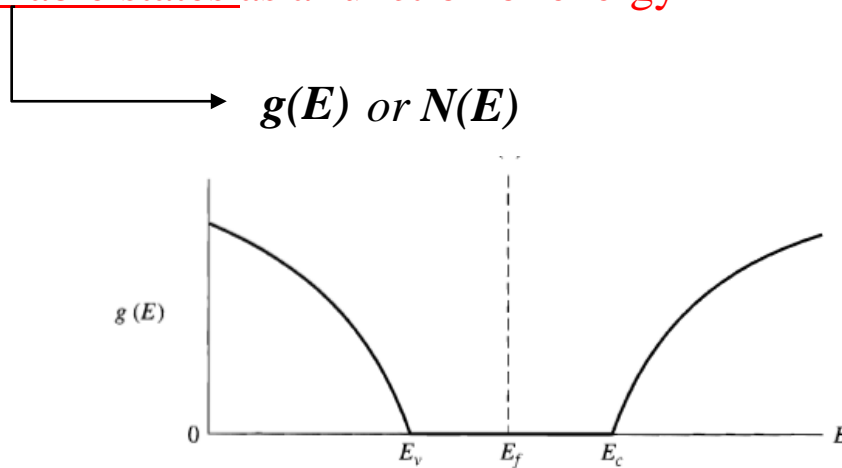


FIGURE 1.10 (b) The density of allowed states for a semiconductor as a function of energy; note that $g(E)$ is zero in the forbidden gap between E_v and E_c .

It is zero in the forbidden gap ($E_c > E > E_v$), but it rises sharply within both the valence band ($E < E_v$) and the conduction band ($E > E_c$). The actual distribution of electrons as a function of energy can be found from the product of the density of allowed states $g(E)$ within a small energy interval dE and the probability $f_D(E)$ that these states are filled.

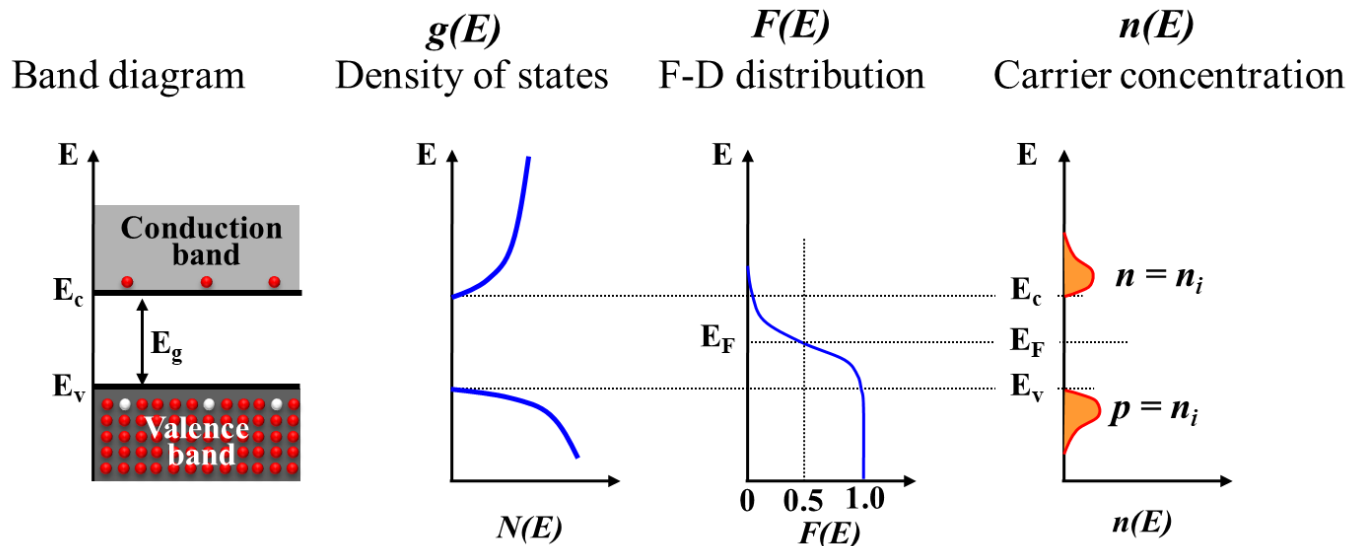
Thermal-Equilibrium Statistics

Carrier concentration in intrinsic semiconductor

The total density of electrons in the conduction band can be obtained by multiplying the density-of-states function $g(E)$ in the conduction band by the Fermi function and integrating over the conduction band

The total density of electrons is strongly related to the carrier concentration [n or $n(E)$]

$$n(E) = g(E) \times F(E)$$



Thermal-Equilibrium Statistics

Carrier concentration in intrinsic semiconductor

Electron concentration in the conduction band

$$n(\mathbf{E}) = g(\mathbf{E}) \times F(\mathbf{E})$$

$$n = \int_0^{\infty} n(\mathbf{E}) d\mathbf{E} = \int_0^{\infty} g(\mathbf{E}) F(\mathbf{E}) d\mathbf{E}$$

$$= \int_0^{\infty} 4\pi \left(\frac{2m_n}{h^2} \right)^{3/2} E^{1/2} \exp[-(E-E_F)/kT] dE$$

$$= \frac{2}{\sqrt{\pi}} N_C (kT)^{-3/2} \int_0^{\infty} E^{1/2} \exp[-(E-E_F)/kT] dE$$

$$g(\mathbf{E}) = 4\pi \left(\frac{2m_n}{h^2} \right)^{3/2} E^{1/2}$$

$$F(\mathbf{E}) = \exp[-(E-E_F)/kT]$$

$$N_C \equiv 2 (2\pi m_n kT/h^2)^{3/2}$$

Effective density of states
→ different for semiconductors

let $x = E/kT$

$$= \frac{2}{\sqrt{\pi}} N_C \exp(E_F/kT) \int_0^{\infty} x^{1/2} e^{-x} dx$$

$$= N_C \exp(E_F/kT)$$

$$= N_C \exp[-(E_C-E_F)/kT]$$

$$\int_0^{\infty} x^{1/2} e^{-x} dx = \frac{\sqrt{\pi}}{2}$$

If we integrate from bottom of conduction band,
 $E \neq 0$, but $E = E_c$.

Thermal-Equilibrium Statistics

Carrier concentration in intrinsic semiconductor

Electron concentration in the conduction band

▪ Electron concentration in the conduction band

$$n = N_c \exp\left[-\frac{(E_c - E_f)}{kT}\right]$$

$N_c \equiv 2 (2\pi m_n kT/h^2)^{3/2}$: effective density of states in CB

$N_c = 2.86 \times 10^{19} \text{ cm}^{-3}$: for Si at RT (300 K)

$N_c = 4.7 \times 10^{17} \text{ cm}^{-3}$: for GaAs at RT (300 K)

▪ Hole concentration in the valence band

$$p = N_v \exp\left[-\frac{(E_f - E_v)}{kT}\right]$$

$N_v \equiv 2 (2\pi m_p kT/h^2)^{3/2}$: effective density of states in VB

$N_v = 2.66 \times 10^{19} \text{ cm}^{-3}$: for Si RT (300 K)

$N_v = 7.0 \times 10^{18} \text{ cm}^{-3}$: for GaAs RT (300 K)

Thermal-Equilibrium Statistics

Carrier concentration in intrinsic semiconductor

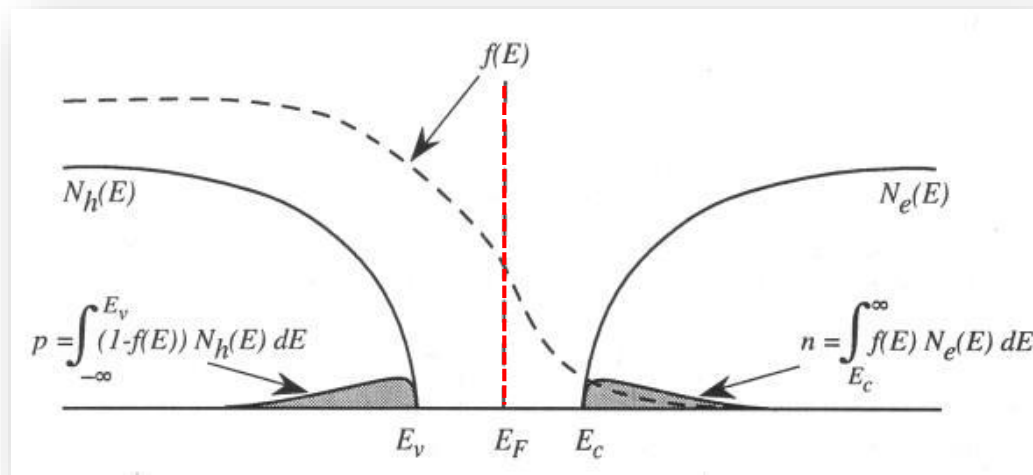
- For intrinsic semiconductors,

$$n = p = n_i$$

$$n = N_c \exp [-(E_c - E_f)/kT] = N_v \exp [-(E_f - E_v)/kT] \quad (\text{multiply both side by } \ln)$$

$$\Rightarrow E_f = E_i = \frac{(E_c + E_v)}{2} + \frac{kT}{2} \ln \left(\frac{N_v}{N_c} \right) \quad kT \sim 0.025 \text{ eV}$$

Middle of the bandgap
Much smaller than E_g at RT (300 K)



\therefore Intrinsic Fermi level (E_f) lies very close to middle of the bandgap

Thermal-Equilibrium Statistics

Carrier concentration in intrinsic semiconductor

When the semiconductor contains a large dopant concentration

$$[N_d \rightarrow N_c \text{ or } N_a \rightarrow N_v (\sim 10^{19} \text{ cm}^{-3} \text{ for Si})],$$

the Fenni-Dirac distribution cannot be approximated by the Maxwell-Boltzmann distribution function.

$$n = n_i \exp\left[\frac{(E_f - E_i)}{kT}\right] \quad p = n_i \exp\left[\frac{(E_i - E_f)}{kT}\right]$$

Above equations are no longer valid, and more exact expressions must be used or the limited validity of the simplified expressions must be realized.

Very highly doped semiconductors ($N_d \geq N_c$, or $N_a \geq N_v$) are called *degenerate* semiconductors because the Fenni level is within the conduction or valence band.

Therefore, allowed states for electrons exist very near the Femi level, just as is the case in metals. Consequently, many of the electronic properties of very highly doped semiconductors *degenerate* into those of metals.

Thermal-Equilibrium Statistics

Carrier concentration in intrinsic semiconductor

- For intrinsic carrier density,

$$n_i = n = p$$

$$n_i^2 = n p$$

$$n_i^2 = N_c N_v \exp [-(E_c - E_f) / kT] \exp [-(E_f - E_v) / kT]$$

$$= N_c N_v \exp [-(E_c - E_v) / kT]$$

$$= N_c N_v \exp [-E_g / kT]$$

$$n_i = \sqrt{N_c N_v} \exp [-E_g / 2kT]$$

smaller $E_g \rightarrow$ more n_i
higher $T \rightarrow$ more n_i

at 300 K

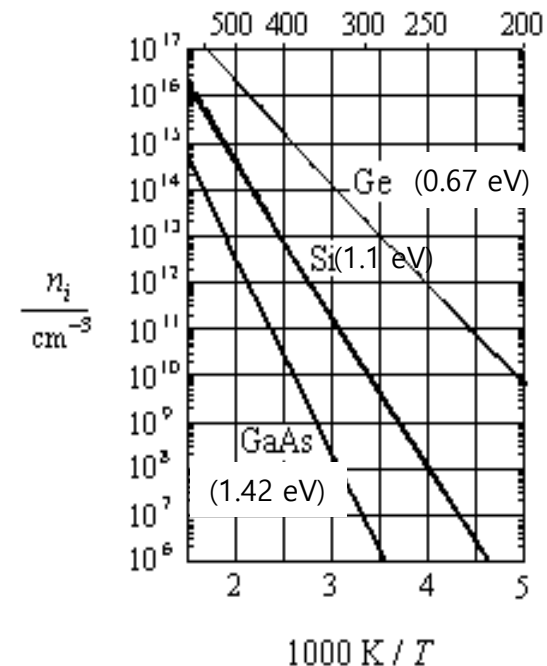
$$\text{Si (1.1 eV)} \quad n_i = 9.65 \times 10^9 \text{ cm}^{-3}$$

$$\text{GaAs (1.42 eV)} \quad n_i = 2.25 \times 10^6 \text{ cm}^{-3}$$

$$n = N_c \exp [-(E_c - E_f) / kT]$$

$$p = N_v \exp [-(E_f - E_v) / kT]$$

Intrinsic carrier density vs Temperature



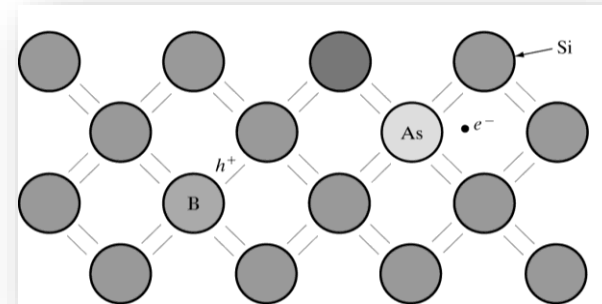
Thermal-Equilibrium Statistics

Carrier concentration in **extrinsic** semiconductor

(1) Donors and Acceptors

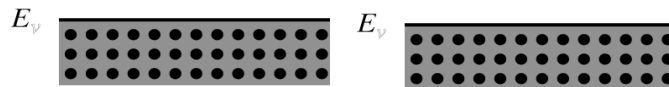
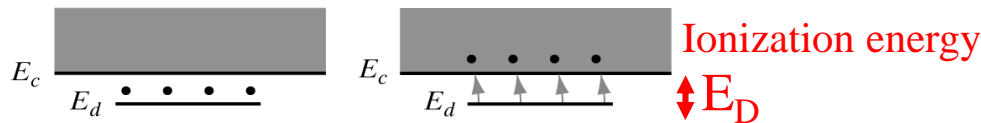
Doped with impurity

- impurity energy levels introduced
- Extrinsic semiconductors



Donor

Generate electron
V elements(Sb, P, As..) for Si

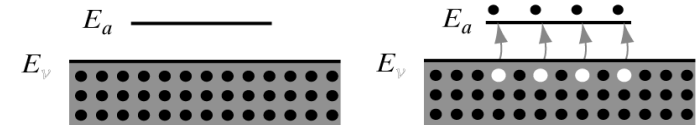


T = 0 K

T = 50 K

Acceptor

Generate hole
III elements(B, Al, Ga..) for Si



T = 0 K

T = 50 K

Thermal-Equilibrium Statistics

Carrier concentration in **extrinsic** semiconductor

- Carrier concentration under **complete ionization condition**

→ RT thermal energy is enough to lead to the full ionization of the dopants

$$\Rightarrow n = N_d, p = N_a$$

Shallow donor concentration N_d

$$n = N_d \quad n = N_c \exp [-(E_c - E_f)/kT]$$

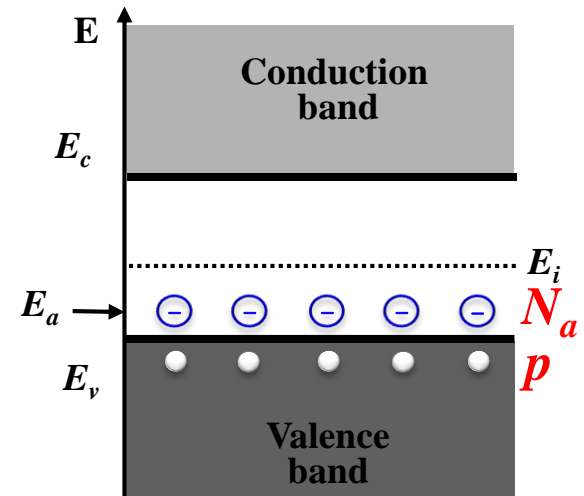
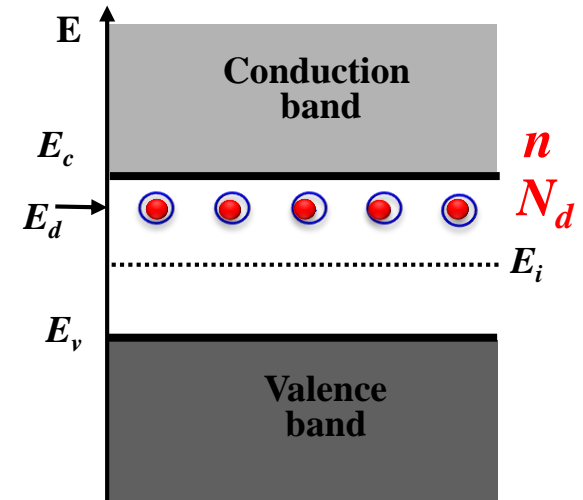
$$E_c - E_f = kT \ln (N_c / N_d)$$

Shallow acceptor concentration N_a

$$p = N_a \quad p = N_v \exp [-(E_f - E_v)/kT]$$

$$E_f - E_v = kT \ln (N_v / N_a)$$

Higher doping concentration (N_d or N_a)
 → E_f move closer to band edge

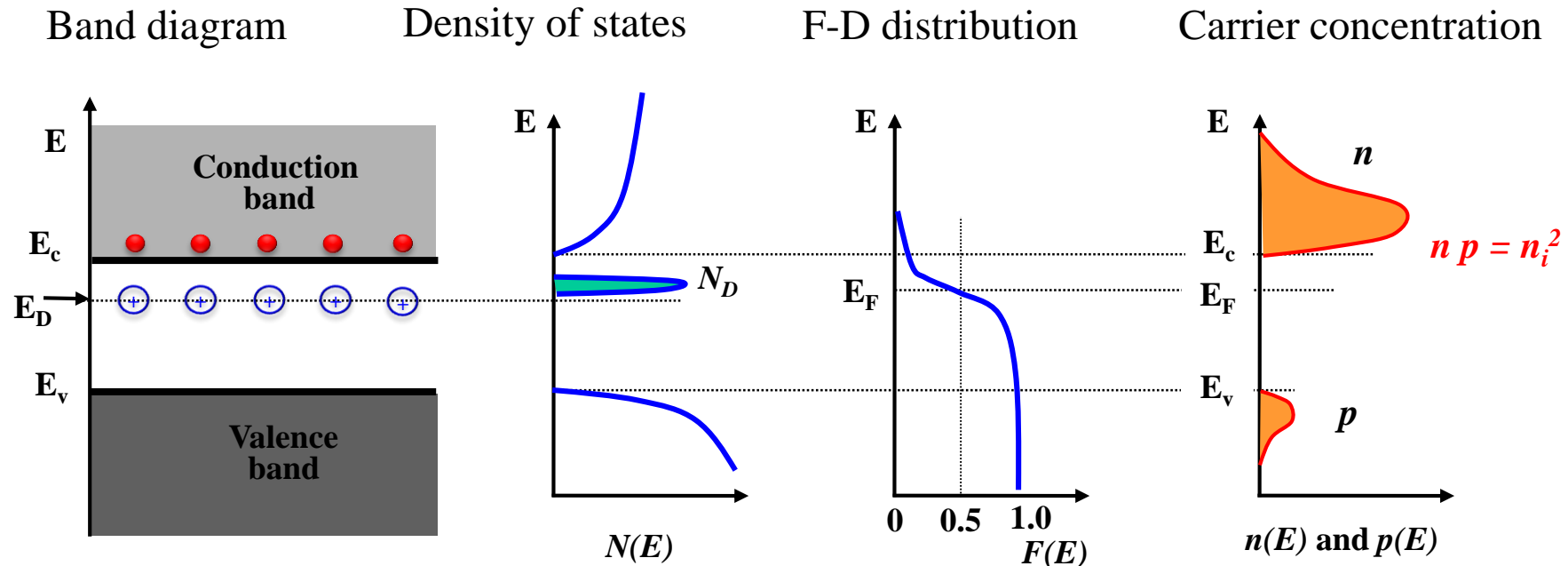


Thermal-Equilibrium Statistics

Carrier concentration in **extrinsic** semiconductor

- Carrier concentration under **complete ionization condition**

For n -type semiconductor



- Electron concentration is much higher than the hole concentration.**
- Fermi level moves toward conduction (valence) band for n -type (p -type).**

Thermal-Equilibrium Statistics

Carrier concentration in **extrinsic** semiconductor

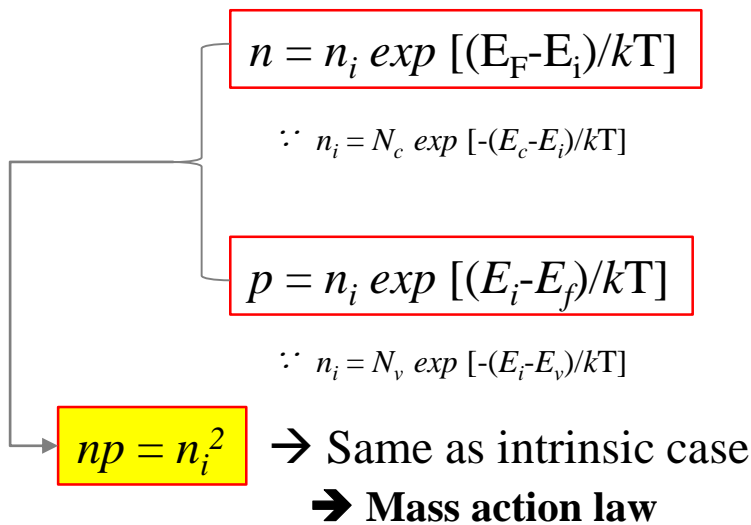
- Carrier concentration under **complete ionization condition**

For *n*-type semiconductor

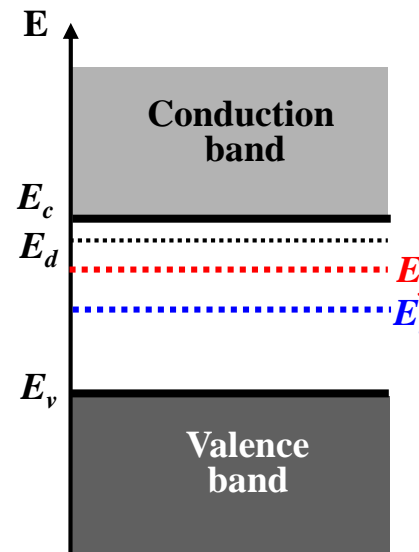
$$n = N_c \exp [-(E_c - E_f)/kT] \quad \text{If we consider reference level of intrinsic Fermi level : } E_i$$

$$n = N_c \exp [-(E_c - E_i)/kT] \exp [(E_f - E_i)/kT]$$

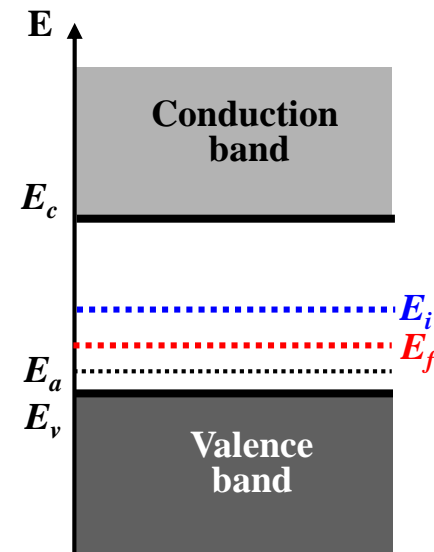
$\underbrace{\hspace{10em}}_{n_i}$



Under thermal equilibrium condition
(i.e., under no heat, light, and electrical field condition)



***n*-type semiconductor**



***p*-type semiconductor**

Thermal-Equilibrium Statistics

Carrier concentration in **extrinsic** semiconductor

- Carrier concentration under **complete ionization condition**

Fermi level adjust itself to preserve charge neutrality

→ total negative charge = total positive charge

$$n + N_a = p + N_d$$

→ under complete ionization condition

N_a : ionized acceptor

N_d : ionized donor

$$np = n_i^2$$



$$n\{n - (N_d - N_a)\} = n_i^2$$



$$n^2 - (N_d - N_a)n - n_i^2 = 0$$

- **Equilibrium electron and hole concentration in *n*-type semiconductor**

$$n = \frac{(N_d - N_a) + \sqrt{(N_d - N_a)^2 + 4 n_i^2}}{2}$$

$$p = n_i^2 / n$$

- **Equilibrium electron and hole concentration in *p*-type semiconductor**

$$p = \frac{(N_a - N_d) + \sqrt{(N_d - N_a)^2 + 4 n_i^2}}{2}$$

$$n = n_i^2 / p$$

Thermal-Equilibrium Statistics

Carrier concentration in **extrinsic** semiconductor

- Majority and minority carrier

If both donor and acceptor impurities are present simultaneously

→ Impurity with larger concentration determines the conductivity type
: **majority carrier**

- majority carrier : electron (hole) in n -type (p -type) → n_n or p_p
- minority carrier : electron (hole) in p -type (n -type) → n_p or p_n

	Majority carrier	Minority carrier
n-type Semiconductor	$n = \frac{(N_d - N_a) + \sqrt{(N_d - N_a)^2 + 4 n_i^2}}{2}$	$p = n_i^2 / n$
p-type Semiconductor	$p = \frac{(N_a - N_d) + \sqrt{(N_d - N_a)^2 + 4 n_i^2}}{2}$	$n = n_i^2 / p$

Thermal-Equilibrium Statistics

Carrier concentration in **extrinsic** semiconductor

- Majority and minority carrier

Generally, in the extrinsic semiconductors (doped semiconductors)

⇒ the net impurity concentration is larger than n_i

⇒ $|N_d - N_a| \gg n_i$

$$n = \frac{(N_d - N_a) + \sqrt{(N_d - N_a)^2 + 4n_i^2}}{2} \approx N_d - N_a \quad (\text{if } N_d > N_a)$$

$$p = \frac{(N_a - N_d) + \sqrt{(N_d - N_a)^2 + 4n_i^2}}{2} \approx N_a - N_d \quad (\text{if } N_a > N_d)$$

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